

**DECLARATION:**

My residence, post office address and citizenship are as stated below next to my name.

[\_] was filed on \_\_\_\_ as Application Serial Number \_\_\_\_ and was amended on \_\_\_\_ (if applicable).

I acknowledge the duty to disclose information which is material to patentability in accordance with Title 37, Code of Federal Regulations, §1.56.

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## Appendix A

### Nonrigid affine correction

One way to estimate a correction matrix  $\mathbf{J} \doteq \mathbf{M} \backslash \tilde{\mathbf{M}}$  generalizes the solution for the rigid affine correction given above. The strategy is to break  $\mathbf{M}$  into column-triples. Each column-triple is a stack of rotation matrices scaled by morph weights. Let  $\mathbf{m}_{f,k,x}^\top, \mathbf{m}_{f,k,y}^\top \in \mathbf{M}$  be the  $x$  and  $y$  projections in frame  $f$  as given by column-triple  $k$ . As in the rigid affine correction, in a properly structured motion matrix  $\mathbf{M}$  these vectors should have equal norm and be orthogonal:

$$\forall_{f,k} \left[ \|\mathbf{m}_{f,k,x}\| = \|\mathbf{m}_{f,k,y}\| \right] \wedge \left[ \mathbf{m}_{f,k,x}^\top \mathbf{m}_{f,k,y} = 0 \right]. \quad (1)$$

Moreover, their projections onto vectors from other column triples should also have equal norm (because all column-triples have the same rotations):

$$\forall_{f,k,j} \left[ \mathbf{m}_{f,k,x} \mathbf{m}_{f,j,x}^\top = \mathbf{m}_{f,k,y} \mathbf{m}_{f,j,y}^\top \right] \wedge \left[ \mathbf{m}_{f,k,x}^\top \mathbf{m}_{f,j,y} = 0 \right]. \quad (2)$$

This yields a system of equations

$$\forall_{f,k,j} \left( \text{vec}(\mathbf{m}_{f,k,x} \mathbf{m}_{f,j,x}^\top - \mathbf{m}_{f,k,y} \mathbf{m}_{f,j,y}^\top) \right)^\top \text{vec} \mathbf{H}_{k,j} = 0, \quad (3)$$

$$\forall_{f,k,j} \left( \text{vec}(\mathbf{m}_{f,k,x} \mathbf{m}_{f,j,y}^\top) \right)^\top \text{vec} \mathbf{H}_{k,j} = 0. \quad (4)$$

Now recall that each  $\mathbf{H}_{k,j}$  is the outer product of two column-triples in  $(\mathbf{J}^{-1})$ , e.g.,

$$\mathbf{H}_{k,j} = (\mathbf{J}^{-1})_{\text{cols}(3k-2, 3k-1, 3k)} (\mathbf{J}^{-1})_{\text{cols}(3j-2, 3j-1, 3j)}^\top. \quad (5)$$

Consequently, the matrix

$$\mathbf{H} \doteq \begin{bmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K,1} & \cdots & \mathbf{H}_{K,K} \end{bmatrix} = (\mathbf{J}^{-1})^{(3K,3)} (\mathbf{J}^{-1})^{(3K,3)\top} \quad (6)$$

should be symmetric with rank 3. Let  $\mathbf{V} \mathbf{\Lambda} \mathbf{V}^\top \xleftarrow{\text{EIG}_3} \mathbf{H}$  be a truncated decomposition of  $\mathbf{H}$  using its three largest eigenvalues and their associated eigenvectors. Then the desired correction is  $(\mathbf{J}^{-1}) = (\mathbf{V} \sqrt{\mathbf{\Lambda}})^{(3K,3)}$ .

Although formally “correct,” this procedure is of limited use because in order to express eqns. (3–4) in terms of  $\mathbf{J}^{-1}$  we must make the substitution  $\mathbf{m}_{f,k,x}^\top \rightarrow \tilde{\mathbf{m}}_{f,x}^\top (\mathbf{J}^{-1})_{\text{cols}(3k-2, 3k-1, 3k)}$ , which makes the constraints on all  $\mathbf{H}_{k,j}$  nearly identical. Consequently the linear system is rank-deficient, because the number of unknowns in  $\mathbf{H}$  grows as  $O(K^4)$  (or  $O(K^3)$  if one only considers  $j = \{k, k+1\}$ ) while the number of true unknowns in  $\mathbf{J}^{-1}$  grows as  $O(K^2)$ . In practice, there are enough constraints to support a usable estimate of the first three columns of  $\mathbf{J}^{-1}$ . We can therefore calculate the first column-triple of  $\tilde{\mathbf{M}}$ , project  $\tilde{\mathbf{M}}$  into the  $3K - 3$  dimensional space orthogonal to this, and repeat the procedure to get the next column triple of  $\tilde{\mathbf{M}}$ . A generalized SVD solution for factoring  $\mathbf{H}$  without explicitly computing its elements (thereby avoiding the rank-deficient division) requires some extra pages to explain and therefore will be published separately.